Name: _____ Roll No: _____

Score: _____ Signature of the Lab Tutor: _____ Date: _____

DEPARTMENT OF ELECTRONIC & TELECOMMUNICATION ENGINEERING
MEHRAN UNIVERSITY OF ENGINEERING & TECHNOLOGY, JAMSHORO
Feedback Control Systems (1st Term, Third Year, 03TL)

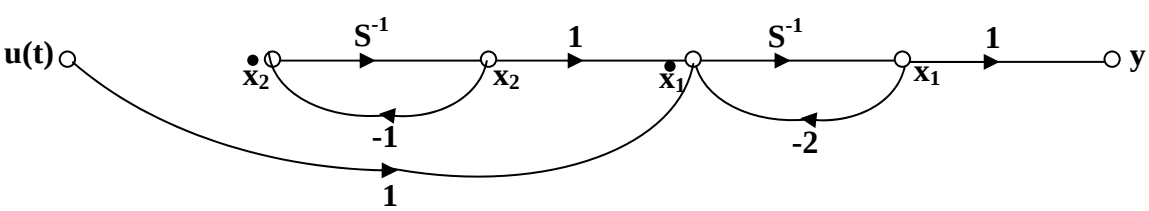
Lab Practice # 11
Controllability & Observability of Linear Systems

1. Introduction

The concepts of controllability and observability play an important role in theoretical and practical aspects of modern control. The conditions on controllability and observability essentially govern the existence of a solution to an optimal control problem. This is the basic difference between optimal control theory and classical control theory. In classical control design for the given set of specifications the designer does not know if any solution exists. The design techniques are dominated by trial-and- error methods. On the other hand optimal control theory has criteria for determining that for given set of specifications the design solution exists or not.

2. Controllability of a linear system

The process is said to be completely controllable if every state variable of the process can be controlled to reach a certain objective in finite time by some unconstrained control u(t). if any one of the state variable is independent of control u(t), there would be no way of driving this particular state variable to a desired state in finite time by means of a control effort. Therefore this particular state is said to be uncontrollable, and as long as there is at least one uncontrollable state the system is not said to be completely controllable or simply uncontrollable. As a simple example of an uncontrollable system following figure shows state diagram of a linear system with two state variables $x_1(t)$ and $x_2(t)$.



Since the control u(t) affects only the state $x_1(t)$, the state $x_2(t)$ is uncontrollable. In other words it would be impossible to drive $x_2(t)$ from an initial state $x_2(t_0)$ to a desired state $x_2(t_f)$ in a finite time interval $t_f - t_0$ by the control u(t). Therefore the entire system is said to be uncontrollable. For the system to be completely controllable it is necessary and sufficient that the controllability matrix S has a rank of n. where n is order of the system. The controllability matrix is given by:

$$\mathbf{S = [B \quad AB \quad A^2B \; . \quad . \quad . \; A^{n-1}B]}$$

In MATLAB we can obtain the controllability matrix S by using built-in function *ctrb.*

Example: - In order to determine the controllability of the system given below

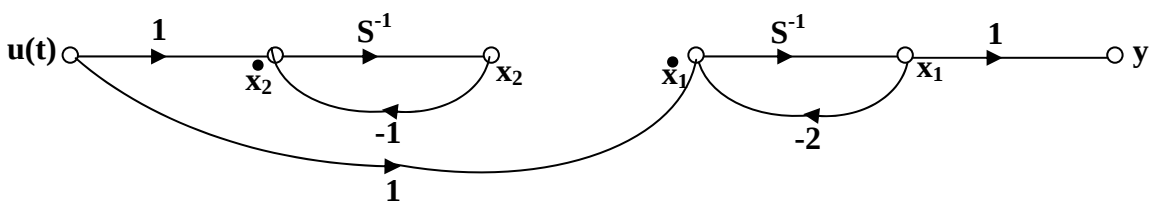
$$\begin{pmatrix} dx_1/dt \\ dx_2/dt \end{pmatrix} = \begin{pmatrix} -5 & -1 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} U$$

We will use following MATLAB code

```
A = [-5  -1; 3   -1];
B = [1  ;  0];
S = ctrb(A,B);
rank(S)
```

3. Observability of a linear system

A system is said to be completely observable if every state variable of the system affects some of the outputs. In other words it is often desirable to obtain information on the state variables from the measurements of the inputs and outputs. If any of the state cannot be observed from the measurement of the outputs the state is said to be unobservable, and the system is not completely observable or simply unobservable. Following figure shows the state diagram of a linear system in which the state x_2 is not connected to the output y(t).



For the system to be completely observable it is necessary and sufficient that the observability matrix V has a rank of n, where n is the order of the system. The observability matrix is given by:

$$V = \begin{pmatrix} C \\ CA \\ CA^2 \\ . \\ . \\ CA^{n-1} \end{pmatrix}$$

In MATLAB we can obtain the observability matrix V by using built-in function *obsv*.

Example: - In order to determine the observability of the system given below

$$\begin{pmatrix} dx_1/dt \\ dx_2/dt \end{pmatrix} = \begin{pmatrix} -5 & -1 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} U$$

$$y = \begin{pmatrix} 1 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

We will use following MATLAB code

```
A = [-5  -1; 3   -1];
C = [1     2];
V = obsv(A,C);
rank(V)
```

Exercise#1: - Determine the controllability and observability of the following systems.

(a)

$$A = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -2 & -3 \\ 0 & 0 & 3 \end{pmatrix} \qquad B = \begin{pmatrix} 3 \\ -6 \\ 3 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \qquad D = 0$$

(b)

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{pmatrix} \qquad B = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \qquad D = 0$$

Exercise#2: -

(a) Determine the controllability of the following systems.

$$G(S) = \frac{S + 2}{S^2 + 7S + 12} \qquad G(S) = \frac{(S+1)(S+2)}{S(S^2+3S+1)}$$

(b) Also determine the observability of the above transfer functions.

Exercise#3: Determine the observability and controllability of sun seeker system.