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DEPARTMENT OF ELECTRONIC & TELECOMMUNICATION ENGINEERING
MEHRAN UNIVERSITY OF ENGINEERING & TECHNOLOGY, JAMSHORO
Feedback Control Systems (1st Term, Third Year, 03TL)

Lab Practice # 05
State Space Representation

1. Introduction

A Control system can be represented as:

$$d/dx = A_x + B_U$$

$$d/dx = C_x + D_U$$

Where A, B, C and D are matrices, x is the state vector, y is the output vector and u is the input vector. The above representation can be obtained from the differential equation or transfer function of the system under considerations.
MATLAB has built-in functions *tf2ss* (transfer function to state space) and *ss2tf* (state space to transfer function) to convert transfer function model into state space representation and vice versa. Following examples demonstrate the use of both of these functions.

Example: - The transfer function of a control system is given below. Find the state space representation of the system.

$$G(S) = \frac{12S + 59}{S^2 + 6S + 8}$$

Use following MATLAB code

```
num = [12  59];
den = [1  6  8];
[A,B,C,D] = tf2ss(num,den);
printsys(A,B,C,D)
```

Exercise#1: - Find the state space representation of the following transfer functions.

$$G(S) = \frac{S + 2}{S^2 + 7S + 12} \qquad G(S) = \frac{(S+1)(S+2)}{S(S^2+3S+1)}$$

Example: - The state space representation of a control system is

$$\begin{pmatrix} dx_1/dt \\ dx_2/dt \end{pmatrix} = \begin{pmatrix} -5 & -1 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} U$$

$$y = \begin{pmatrix} 1 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

The transfer function of the system can be obtained as

```
A = [-5   -1;3   -1]
B = [1 ;  0];
C = [1   2];
D = [0];
[num,den]= ss2tf(A,B,C,D);
printsys(num,den);
```

Exercise#2: - Find the transfer function of the following systems.

(a)

$$A = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -2 & -3 \\ 0 & 0 & 3 \end{pmatrix} \qquad B = \begin{pmatrix} 3 \\ -6 \\ 3 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \qquad D = 0$$

(b)

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{pmatrix} \qquad B = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \qquad D = 0$$

2. Eigen Values

The eign values are the roots of the denominator polynomial of a transfer function. The built function *eig(A)* computes the eign values of the system.

Exercise#3: Find the eign values of the system with

(a)

$$A = \begin{pmatrix} 0 & 2 & 0 & 0 & 0 \\ 0.1 & 0.35 & 0.1 & 0.1 & 0.75 \\ 0 & 0 & 0 & 2 & 0 \\ 0.4 & 0.4 & -0.4 & -0.4 & 0 \\ 0 & -0.03 & 0 & 0 & -1 \end{pmatrix}$$

(b) Compute the eign values of the system given in exercise 2(a) and 2(b).

3. State Transition Matrix

The state transition matrix $\Phi(t)$ can be computed using the built function *expm(A*t)*.

Exercise#4: - Find the state transition matrix of the following systems

(a) System given in exercise 2(a) at t = 4
(b) System given in exercise 2(b) at t = 0